- Sums involving even and odd sequences have the following properties:
  - The sum of two even sequences is even.
  - The sum of two odd sequences is odd.
  - The sum of an even sequence and odd sequence is neither even nor odd, provided that neither of the sequences is identically zero.
- That is, the *sum* of sequences with the *same type of symmetry* also has the *same type of symmetry*.
- Products involving even and odd sequences have the following properties:
  - The product of two even sequences is even.
  - The product of two odd sequences is even.
  - The product of an even sequence and an odd sequence is odd.
- That is, the *product* of sequences with the *same type of symmetry* is *even*, while the *product* of sequences with *opposite types of symmetry* is *odd*.

- Every sequence x has a *unique* representation of the form

$$x(n) = x_e(n) + x_o(n),$$

  where the sequences x_e and x_o are *even* and *odd*, respectively.
- In particular, the sequences x_e and x_o are given by

$$x_e(n) = \tfrac{1}{2}[x(n) + x(-n)] \quad \text{and} \quad x_o(n) = \tfrac{1}{2}[x(n) - x(-n)].$$

- The sequences x_e and x_o are called the even part and odd part of x, respectively.
- For convenience, the even and odd parts of x are often denoted as $\mathrm{Even}\{x\}$ and $\mathrm{Odd}\{x\}$, respectively.

- The least common multiple of two (strictly positive) integers a and b, denoted $\operatorname{lcm}(a, b)$, is the smallest positive integer that is divisible by both a and b.
- The quantity $\operatorname{lcm}(a, b)$ can be easily determined from a prime factorization of the integers a and b by taking the product of the highest power for each prime factor appearing in these factorizations. Example:

$$\operatorname{lcm}(20, 6) = \operatorname{lcm}(2^2 \cdot 5^1, 2^1 \cdot 3^1) = 2^2 \cdot 3^1 \cdot 5^1 = 60;$$

$$\operatorname{lcm}(54, 24) = \operatorname{lcm}(2^1 \cdot 3^3, 2^3 \cdot 3^1) = 2^3 \cdot 3^3 = 216; \quad \text{and}$$

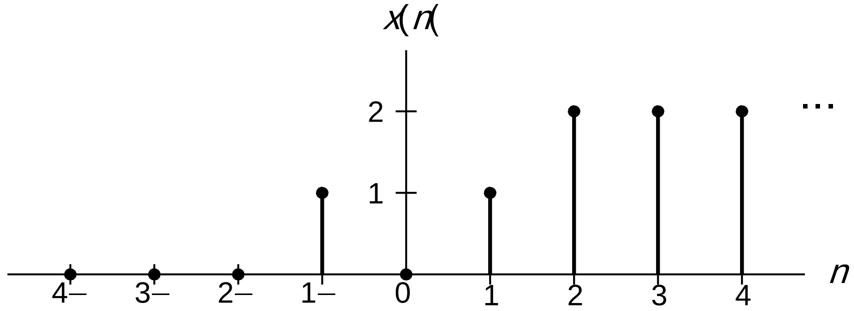
$$\operatorname{lcm}(24, 90) = \operatorname{lcm}(2^3 \cdot 3^1, 2^1 \cdot 3^2 \cdot 5^1) = 2^3 \cdot 3^2 \cdot 5^1 = 360.$$

- **Sum of periodic sequences.** For any two periodic sequences x_1 and x_2 with fundamental periods N_1 and N_2, respectively, the sum $x_1 + x_2$ is *periodic* with period $\operatorname{lcm}(N_1, N_2)$.

- A signal x is said to be right sided if, for some (finite) integer constant n_0, the following condition holds:

$$x(n) = 0 \quad \text{for all } n < n_0$$

(i.e., x is *only potentially nonzero to the right of* n_0).
- An example of a right-sided signal is shown below.



- A signal x is said to be causal if

$$x(n) = 0 \quad \text{for all } n < 0.$$

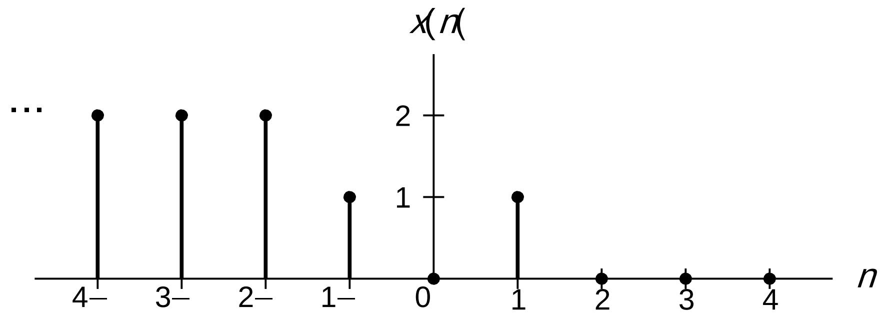
- A causal signal is a *special case* of a right-sided signal.
- A causal signal is not to be confused with a causal system. In these two contexts, the word "causal" has very different meanings.

- A signal x is said to be left sided if, for some (finite) integer constant n_0, the following condition holds:

$$x(n) = 0 \quad \text{for all } n > n_0$$

)i.e., x is *only potentially nonzero to the left of* n_0).

- An example of a left-sided signal is shown below.

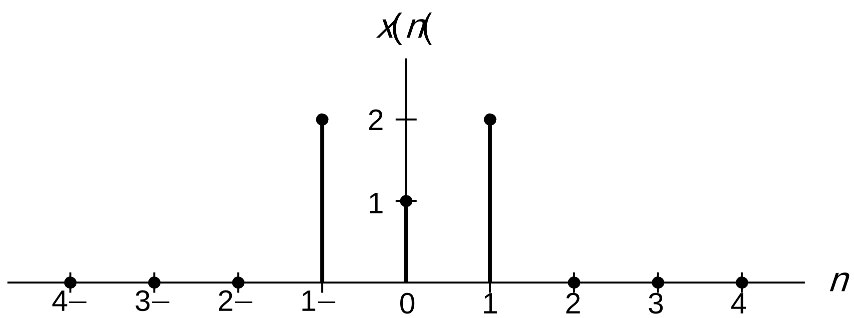


- A signal x is said to be anticausal if

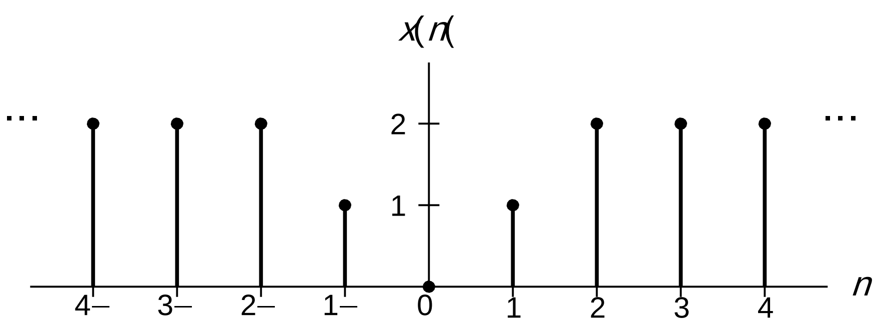
$$x(n) = 0 \quad \text{for all } n \geq .0$$

- An anticausal signal is a *special case* of a left-sided signal.
- An anticausal signal is not to be confused with an anticausal system. In these two contexts, the word “anticausal” has very different meanings.

- A signal that is both left sided and right sided is said to be finite duration)or time limited.(
- An example of a finite-duration signal is shown below.



- A signal that is neither left sided nor right sided is said to be two sided.
- An example of a two-sided signal is shown below.



- A signal x is said to be bounded if there exists some (*finite*) positive real constant A such that

$$|x(n)| \leq A \quad \text{for all } n$$

(i.e., $x(n)$ is *finite* for all n).

- Examples of bounded signals include any constant sequence.
- Examples of unbounded signals include any nonconstant polynomial sequence.

- The energy E contained in the signal x is given by

$$E = \sum_{k=-\infty}^{\infty} |x(k)|^2 .$$

- A signal with finite energy is said to be an energy signal.

Section 7.3

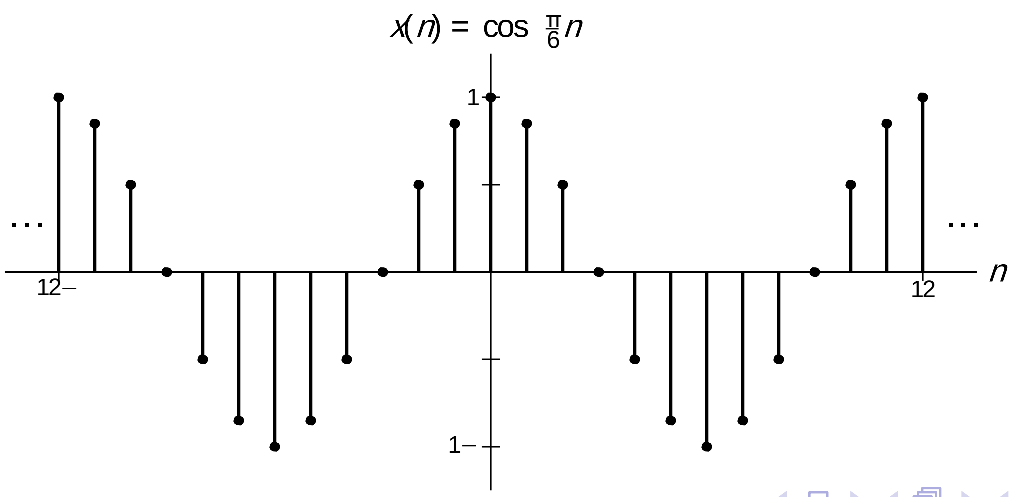
Elementary Signals

- A (DT) real sinusoid is a sequence of the form

$$x(n) = A\cos(\Omega n + \theta),$$

where A, Ω, and θ are *real* constants.

- A real sinusoid is *periodic* if and only if $\frac{\Omega}{2\pi}$ is a *rational number*, in which case the fundamental period is the *smallest integer* of the form $\frac{2\pi k}{|\Omega|}$ where k is a positive integer.
- For all integer k, $x_k(n) = A\cos([\Omega + 2\pi k]n + \theta)$ is the *same* sequence.
- An example of a periodic real sinusoid with fundamental period 12 is shown plotted below.



- A (DT) complex exponential is a sequence of the form

$$x(n) = ca^n,$$

where c and a are *complex* constants.

- Such a sequence can also be equivalently expressed in the form

$$x(n) = ce^{bn},$$

where b is a *complex* constant chosen as $b = \ln a$. (This this form is more similar to that presented for CT complex exponentials).

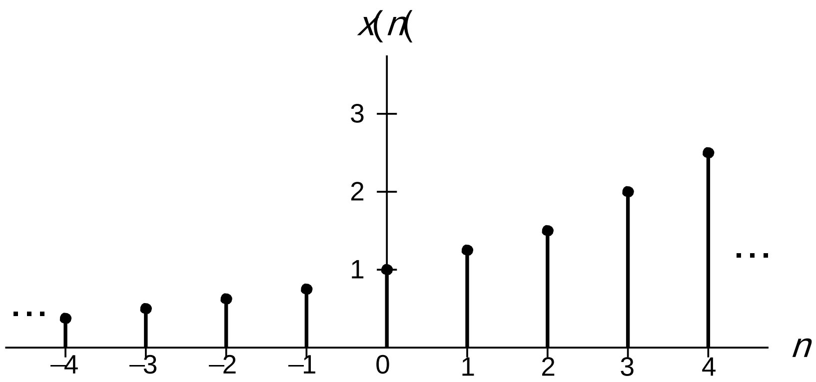
- A complex exponential can exhibit one of a number of *distinct modes of behavior*, depending on the values of the parameters c and a.
- For example, as special cases, complex exponentials include real exponentials and complex sinusoids.

- A (DT) real exponential is a special case of a complex exponential

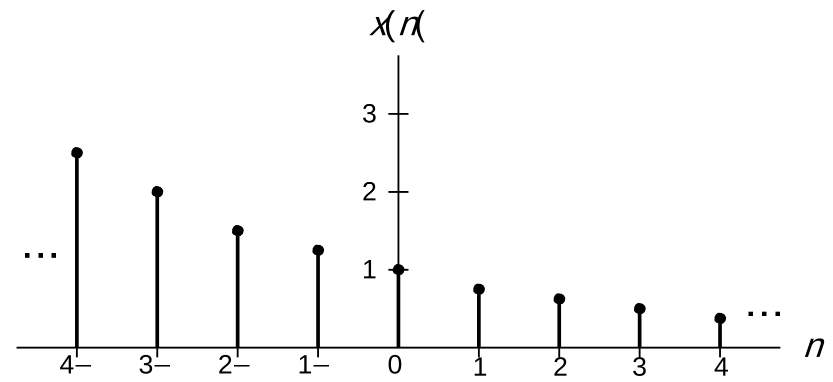
$$x(n) = ca^n,$$

where c and a are restricted to be *real* numbers.

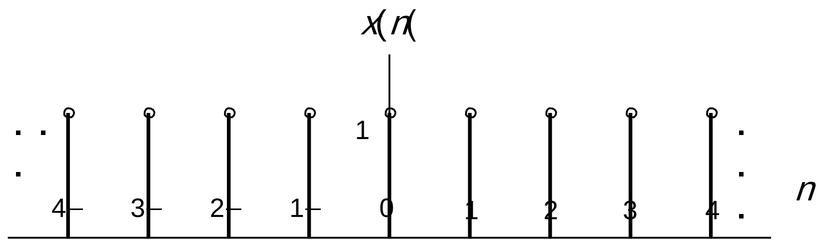
- A real exponential can exhibit one of *several distinct modes* of behavior, depending on the magnitude and sign of a.
- If $|a| > 1$, the magnitude of $x(n)$ *increases* exponentially as n increases (i.e., a growing exponential).
- If $|a| < 1$, the magnitude of $x(n)$ *decreases* exponentially as n increases (i.e., a decaying exponential).
- If $|a| = 1$, the magnitude of $x(n)$ is a *constant*, independent of n. If
- $a > 0$, $x(n)$ has the *same sign* for all n.
- If $a < 0$, $x(n)$ *alternates in sign* as n increases/decreases.



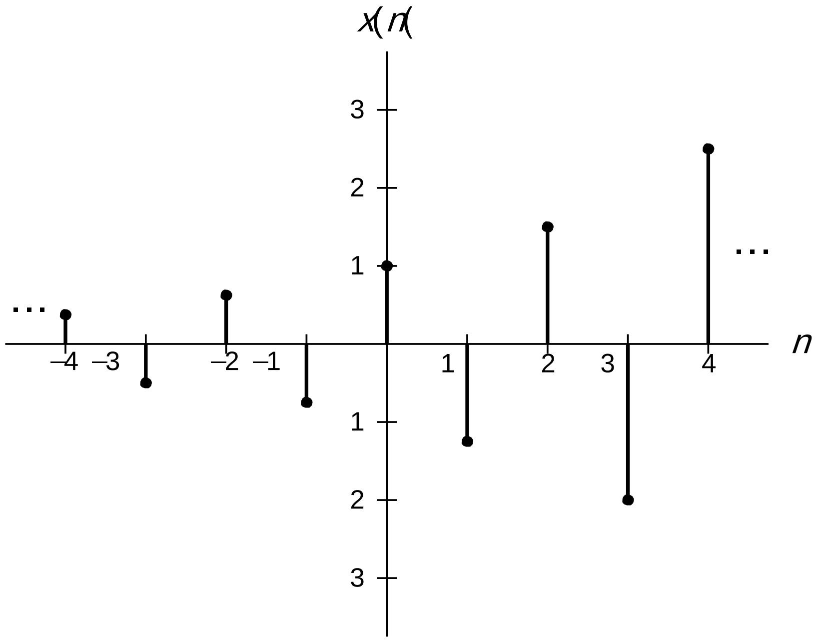
$|a| > 1,\ a > 0$]$a = \frac{5}{4}$; $c =$ [1



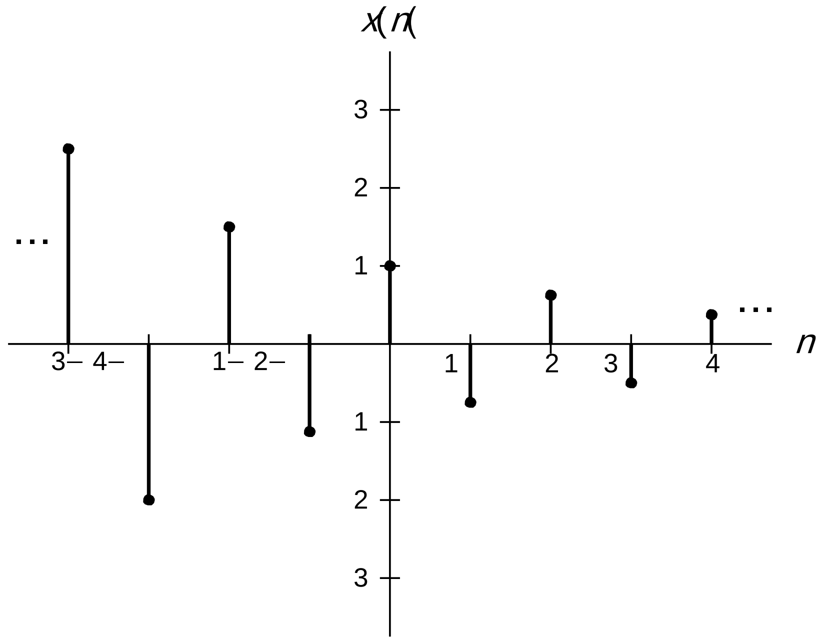
$|a| < 1,\ a > 0$]$a = \frac{4}{5}$; $c =$ [1



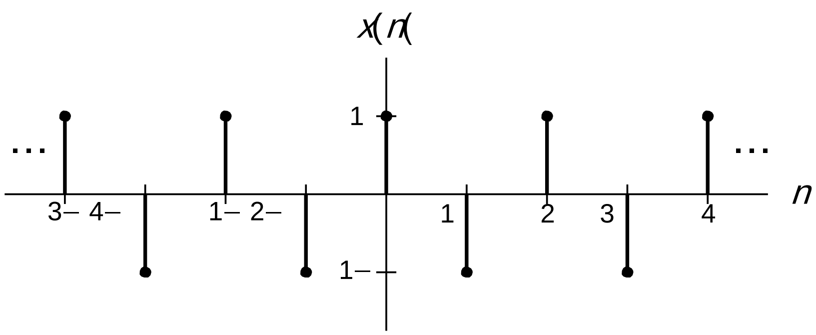
$|a| = 1,\ a > 0$]$a = 1$; $c =$ [1



$|a| > 1,\ a < 0$]$a = -\frac{5}{4}$; $c =$ [1



$|a| < 1,\ a < 0$]$a = -\frac{4}{5}$; $c =$ [1



$|a| = 1,\ a < 0$]$a = -1$; $c =$ [1

- A complex sinusoid is a special case of a complex exponential $x(n) = ca^n$, where c and a are *complex* and $|a| = 1$ (i.e., a is of the form $e^{j\Omega}$ where Ω is real).
- That is, a (DT) complex sinusoid is a sequence of the form

$$x(n) = ce^{j\Omega n},$$

where c is *complex* and Ω is *real*.
- Using Euler's relation, we can rewrite $x(n)$ as

$$x(n) = \underbrace{|c|\cos(\Omega n + \arg c)}_{\operatorname{Re}\{x(n)\}} + j\underbrace{|c|\sin(\Omega n + \arg c)}_{\operatorname{Im}\{x(n)\}}.$$

- Thus, $\operatorname{Re}\{x\}$ and $\operatorname{Im}\{x\}$ are real sinusoids.
- A complex sinusoid is *periodic* if and only if $\frac{\Omega}{2\pi}$ is a *rational number*, in which case the fundamental period is the *smallest integer* of the form $\frac{2\pi k}{|\Omega|}$ where k is a positive integer.